Going for Broke: Restructuring Distressed Debt Portfolios [1]

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Abstract

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This paper discusses how to restructure a portfolio of distressed debt, what the gains are from doing so, and attributes these gains to restructuring and portfolio effects. This is an interesting and novel problem in fixed-income portfolio management that has received scant modeling attention. We show that debt restructuring is Pareto improving and lucrative for borrowers, lenders, and investors in distressed debt. First, the methodological contribution of the paper is a parsimonious model for the pricing and optimal restructuring of distressed debt, i.e., loans that are under-collateralized and are at risk of borrower default, where willingness to pay and ability to pay are at issue. Distressed-debt investing is a unique portfolio problem in that (a) it requires optimization over all moments, not just mean and variance, and (b) with debt restructuring, the investor can endogenously alter the return distribution of the candidate securities before subjecting them to portfolio construction. Second, economically, we show that post-restructuring return distributions of distressed debt portfolios are attractive to fixed-income investors, with risk-adjusted certainty equivalent yield pick-ups in the hundreds of basis points, suggesting the need for more efficient markets for distressed debt, and shedding light on the current policy debate regarding the use of eminent domain in mitigating real estate foreclosures.

Keywords: distressed debt, restructuring, pooling, debt overhang, eminent domain.
JEL codes: G11, G13, G33

1 Introduction

The last decade was characterized by over-leveraging in many credit sectors, including sovereign, municipal, corporate, credit card, and mortgage debt. Volatile and correlated underlying collateral resulted in an eventual collapse of many of these debt markets. In the case of corporate high-yield debt, 17.9% was classified as distressed in 2011, as opposed to only 7.9% a year earlier (Altman and Kuehne (2012)). In the case of sovereign debt, "...sovereign economic conditions appear to deteriorate relatively frequently. Often this requires massive debt restructurings and/or bailouts accompanied by painful austerity programs" (Altman and Rijken (2011)). Similarly, the Home Affordable Modification Program (HAMP) reports that it has supported the restructuring of 1.3 million first and second lien mortgage loans, short sales, deed-in-lieu transactions, and forbearance plans.[1] Thus, distress in diverse debt portfolios has given rise to a unique fixed-income portfolio management problem that we address in this paper.

In this paper, we demonstrate how to optimize the restructuring and portfolio construction of pools of distressed debt, deriving the pre- and post-restructuring values of underwater loans. Situations involving possible strategic or liquidity defaults benefit from restructuring relief, whereby the loan is modified to prevent default in a way that enhances the economic value of the loan for the investor. Indeed, the restructuring envisaged in this paper shows that eliminating the deadweight costs of default is Pareto-improving, with gains that may be shared by the original lender, borrower, and investor, assuming a fair price at which distressed loans are taken over. Overall, we show that the gains from restructuring are potentially large enough that there is enough to go around to all parties in distressed loan markets.

Managing a portfolio of distressed debt starts with identifying the source of distress. Distressed debt suffers from two major sources of concern to investors. For one, "liquidity" default may occur, whereby the borrower is willing but unable to make payments on the loan. Alternatively, "strategic" default may occur, whereby solvent borrowers are unwilling to continue making payments on their loans because the collateral is deeply underwater.[2]

[1] See: `http://www.treasury.gov/initiatives/financial-stability/reports/Documents/October%202012%20MHA%20Report%20Final.pdf`.

[2] For an interesting empirical approach to distinguishing these two types of default, see Giroud, Mueller,

Such borrowers are more likely to exercise their put option to default,[3] walking away from their loans by putting the depressed collateral back to the lender. The economic significance of strategic default has been theoretically modeled (e.g., Anderson and Sundaresan (1996); Mella-Barral and Perraudin (1997)), and empirically documented (e.g., Ghent and Kudlyak (2011); Guiso, Sapienza, and Zingales (2011)).

The relation between debt value and its collateral is direct: a decline in collateral value results in a reduced willingness to pay, thereby driving strategic default. The problem may also work in reverse, where over-leveraging can have an impact on underlying collateral value, since the *debt overhang* problem results in a failure to invest in the collateral, precipitating a downward spiral in debt value.[4]

We show that deleveraging by writing down debt is a first step in optimal restructuring of a distressed debt portfolio. Overcoming resistance to principal write-downs is key in a restructuring as it also prevents firms from being saddled with excess leverage that often necessitates subsequent costly restructuring (Gilson (1995)).[5] Asquith, Gertner, and Scharfstein (1994) examine the effectiveness of several restructuring approaches, yet the approach advocated here, i.e., principal write-downs, seems less favored, even though it is shown to be optimal compared to alternative approaches such as rate reductions, see Das (2012).

Whereas much of the literature has focused on the negative outcomes of over-leveraging, this paper provides normative prescriptions for ameliorating these problems through deleveraging, and an analysis of the risks and returns that accrue to the original lenders as well as the financial intermediaries who can purchase and optimally restructure distressed debt. Prior work on debt restructuring in the mortgage markets has presented economic arguments for loan principal modification (Das (2012)) as well as closed-form solutions for distressed debt pricing (Das and Meadows (2013)). Here, we extend the model in the latter paper to optimize restructuring and portfolio construction of pools of distressed debt, and we explore

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Stomper, and Westerkamp (2012).

[3]See Merton (1974) for the original analysis of corporate debt, and more recently, Gapen, Gray, Lim, and Xiao (2008) and Gray and Jobst (2011) for sovereign debt.

[4]Debt overhang results in underinvestment by firms (Myers (1977); Lang, Ofek, and Stulz (1996)), suboptimal public projects by countries (Krugman (1988)), failure by individuals to upkeep a home (Melzer (2010).

[5]It is not surprising that lenders resist principal write downs even when shown to be optimal. Ghent (2011) presents evidence of reluctance in principal forgiveness in her sample from the Great Depression.

the gains accrued from special features such as equity-sharing and debt-equity swaps. We also explore how much of the investor's gain emanates from restructuring itself and how much from portfolio construction. In addition, we note that the gains from restructuring are extremely large, representing frictions from the market's failure to clear distressed debt markets, even after assuming that unrestructured debt is purchased from lenders at fair market value.

Investing in portfolios of distressed debt involves divergence from standard portfolio construction paradigms in two ways. First, the return distribution of distressed debt itself depends on restructuring by the investor, and is no longer exogenous. The owner of distressed debt is allowed to modify the terms of the loan to stave off default, and thereby alter the distribution of returns with which portfolio construction is undertaken.[6] Therefore, the gains to constructing portfolios of distressed debt come from: (a) the adjustments and restructuring of individual loans by the investor, and (b) the diversification and optimal portfolio construction across all loans. We analyze which of these sources generates more value. Second, when portfolios involve equities and high-quality bonds, traditional Markowitz (1952) mean-variance optimization is applied because the individual and joint distributions of returns are assumed to be approximately univariate/multivariate Gaussian. In contrast, the distribution of returns of distressed debt is highly non-Gaussian, and the mean-variance paradigm is no longer applicable. Therefore, our analysis is cognizant of higher-order moments of risk such as skewness and kurtosis of returns.

In this paper, we examine the gains from buying distressed debt, and optimally constructing restructured debt portfolios. Our model in this paper proposes a reduced-form specification that captures the borrower's ability to pay as well as his willingness to pay, measured by a function of the debt principal relative to collateral value, and our recommended approach will make higher-moment risk-adjusted comparisons of optimally restructured debt versus distressed debt before restructuring. We enumerate the main results of the paper here:

1. *Higher-moment risk-adjusted restructuring gains*: (i) The gains from restructuring a single loan are substantial, and run into hundreds of basis points in certainty equiva-

[6]This second feature is not specific to distressed debt. Debt is renegotiated even in absence of covenant violations or distress [Roberts and Sufi (2009)]. Covenants themselves are also renegotiated prior to violation, indicating that lenders take an active role in monitoring a firm's investments in a way that decreases likelihood of default.

lence for a constant relative risk aversion (CRRA) investor. (ii) Restructuring results in higher expected returns and lower standard deviation, and hence, offers debt holders a substantial gain even in mean-variance space. (iii) These gains are greater for loans that are more deeply underwater, and remain large after accounting for the high skewness and kurtosis in returns that persist even after restructuring.

2. *Gains from shared equity restructuring*: We show that restructuring using shared-equity-appreciation loans (i.e., SEALs, wherein the lender/investor offers a principal write-down in exchange for a share of the collateral price appreciation), offer higher risk-adjusted rates of return to investors. SEALs can take the form of shared-appreciation mortgages (SAMs) in the case of mortgage debt, equity exchange offers in the case of corporate debt, or debt-equity swaps in the case of sovereign debt.

3. *Dependence of gains on recovery rates*: The gains from restructuring increase with the default discount (i.e., the liquidity discount plus costs on forced or liquidation sales), and remain substantial even under modest default discounts of 10%–30%.

4. *Pooling and portfolio construction*: We examine investor benefits to pooling distressed loans. (i) We find that the benefits of diversification manifest through reduced standard deviation risk, skewness risk, and kurtosis risk when combining distressed loans into portfolios. (ii) We find that the investor gains from diversification are larger for unrestructured loan pools than for restructured pools. (iii) Overall, risk-adjusted gains from investing in portfolios of distressed loans come mainly from optimal restructuring of individual loans and to a lesser extent from diversification.

5. *Managing ability-to-pay*: (i) Restructuring using principal write-downs also mitigates the default risk from lack of ability to pay. (ii) Restructuring using rate reductions results in additional gains over principal write-downs when there are substantial ability-to-pay concerns. (iii) Restructuring with rate relief but no principal relief does not offer risk-adjusted portfolio gains when the investor's willingness to pay is the chief concern.

Given the range of benefits from restructuring via principal write downs, and the large gains available for sharing, eminent domain may be a practical way of unleashing benefits that are otherwise mired in potential foreclosure processes.[7] [8]

In related work, empirical studies have examined the factors driving distressed-debt portfolio renegotiations, and the success of such activity.[9] For instance, Gilson, John, and Lang (1990) and Gilson (1991) find that firms with successful renegotiations outside the court process realized a 30% gain in stock values, and those that failed to do so, experienced similar stock declines. Asquith, Gertner, and Scharfstein (1994) examine the prevalence and effectiveness of several restructuring approaches, finding that principal write-downs are a less favored method of banks; though, James (1996) finds that the success of debt restructuring through exchange offers is crucially related to and depends on whether banks forgive principal. Agarwal, Amromin, Ben-David, Chomsisengphet, and Evanoff (2010) find that a one percent relief in mortgage interest rates, results in a four percent decline in defaults. Nonetheless, the risk-return tradeoffs to investors under different restructuring plans and the normative prescriptions thereof remain unexplored. This paper shows that gains from restructuring are large enough that all parties stand to gain from a fairly executed loan modification. Therefore, implementing restructuring (via eminent domain or otherwise) has the potential to enrich investors while assuring lenders and borrowers more than their current values.

Overall, the range of investment strategies in distressed debt is large, and the key at-

[7]Critiques of forced restructuring via eminent domain abound. Whether such a use of eminent domain is constitutional has yet to be clarified in the courts. Moreover, the practice has the potential to scare away future lenders and investors in mortgage securities, putting the long-term recovery of distressed cities at risk. Banks would be forced to recognize losses more swiftly, impacting their continued participation in lending markets. In addition, such plans may favor rash borrowers by transforming their underwater mortgages to positive equity at the expense of prudent borrowers and taxpayers.

[8]Nonetheless, the application of eminent domain has several advantages. Distressed debt is characterized by high rates of default, high loss given default (LGD), and consequently high standard deviations and negative skewness in return distributions. Thus, absent restructuring and related renegotiation, these distressed securities are unattractive investments for fixed-income investors. "Own" (non-securitized) loans are easy to restructure through bilateral negotiation between lender and borrower, but securitized loans, with distributed ownership, are not. Acquisition via eminent domain is one feasible solution to initiate restructuring. Clearing the logjam of distressed debt by forced restructuring saves deadweight foreclosure costs, and enhances property values in towns like Richmond, better preserving the tax base. Default rates are lowered, keeping homeowners in their homes, and investors such as pension funds attain safer portfolios.

[9]Alternatives to restructuring are liquidation, refinancing, and repayment plans. See Agarwal, Amromin, Ben-David, Chomsisengphet, and Evanoff (2010) for an empirical analysis of the alternatives. Piskorski, Seru, and Vig (2010) find that securitized mortgage loans are less likely to be restructured than bank-held debt, and therefore have greater foreclosure rates.

traction of this asset class is that investment opportunities are counter-cyclical to the broad economy (DePonte (2010)), which makes it a useful risk diversification vehicle in standard portfolios and an attractive asset class to a broad range of institutional investors, such as hedge funds and private equity investors, who can purchase and restructure these loans in an attempt to earn favorable risk-adjusted returns in periods of economic downturn.[10] In the past two decades leading up to the end of 2010, the HFRI Distressed / Restructuring Index delivered 12.7% annualized returns to investors; in comparison, the S&P 500 delivered 8.0%.[11]

The paper proceeds as follows: In Section 2 we describe a tree-based model for the pricing of distressed debt with coupons and default risk.[12] We impose default using a barrier function that considers incentive effects from shared-appreciation and willingness to pay, and other frictions. This approach is driven by practical considerations that make the model easier to implement and account for non-optimal default behavior. In Section 3 we examine various unique return features of distressed debt with this model for investments of typical horizons, usually shorter than the maturity of the original loan. We also show how restructuring results in a vast improvement in return characteristics, converting an erstwhile unattractive loan into one that has attractive value in a distressed fixed-income portfolio.

In Section 4 we show how optimal restructuring of distressed debt may be assessed by examining the expected utility of an investor with CRRA preferences, i.e., a utility function that is impacted by higher-order moments and not just mean and variance. The gains from restructuring are converted into a yield pick up that is in the hundreds of basis points. In Section 5, we discuss extensions and conclude.

[10]There are a myriad of strategies used for investments in distressed debt, also known as "vulture investing". See Gilson (1995) for an early survey.

[11]www.gramercy.com, "Distressed Debt Investing – An Overview," August 2010.

[12]This model embeds and generalizes the closed-form approach developed in Das and Meadows (2013) where willingness to pay by the borrower (a proxy for the hazard rate of default) is implicitly defined and modulates the likelihood of default.

2 Modeling Distressed Debt

We employ a discrete-time, finite horizon model for distressed debt that is similar to the continuous-time model of Black and Cox (1976) but is enhanced to accommodate coupons and sharing of equity in the underlying asset by borrower and lender. The model's binomial tree structure is simple to implement, and allows for richer features such as coupons and exogenous default barriers.

Basic Notation: We assume that the face value of the debt/loan is L with maturity/horizon T (in years). The collateral for this debt are assets/holdings of the borrower, denoted H, and default is triggered when H drops to a default barrier D specified by a function explained in detail subsequently. On default, the lender receives a recovery fraction, $\phi \in [0, 1]$, of the collateral. Recovery rates vary by type of debt. (i) Campbell, Giglio, and Pathak (2008) find that the discount on real estate foreclosure is around 32% in Massachusetts, which is at the higher end of the scale for sales of bank-owned properties, also known as REO sales (see also Lee and Kai-yan (2010)). In other states, the lower end of the REO discount is around 10%. (ii) Recovery rates can be even lower in the corporate sector. According to Moody's, the average recovery rate for senior unsecured bonds, measured by post-default trading prices, rose from 37.1% in 2009 to 49.5% in 2010. (iii) Average historical recovery on sovereign debt is 53%.[13] Fire sale discounts are potentially large, see Coval and Stafford (2007).

Underlying collateral: To model collateral/asset value H, we assume a geometric Brownian motion, i.e.,

$$dH_t = \mu H_t \, dt + \sigma H_t \, dZ_t \tag{1}$$

with drift μ, volatility coefficient σ, and scalar Wiener process Z_t. Discounting is undertaken at risk-free rate r for default-adjusted cash flows in this risk-neutral option-pricing framework.

Default barrier: Debt is distressed when H, the value of the underlying assets, approaches a default barrier D. This is a standard approach in credit models, as in Merton (1974), Black and Cox (1976), Finger (2002), Lando (2004). The barrier is a function of the loan amount

[13] Moody's Investor Service, "Corporate Default and Recovery Rates, 1920–2010," Special Comment. Moody's 2011 Sovereign Default Study. `www.moodys.com`.

L, and we define it using the following function, adopted from Das and Meadows (2013):

$$D = L \ \exp[-\gamma(1-\theta)], \tag{2}$$

where θ denotes an equity stake in the underlying assets when distressed debt is restructured using a debt-equity swap. If the underlying assets eventually grow in value and the debt appreciates, the equity stake receives a fraction θ of assets H above a strike K. The more upside ($\theta < 1$) the borrower gives up, the greater the incentive to default and bequeath the distressed assets to the lender, and the higher the default barrier. The parameter $\gamma > 0$ specifies the borrower's willingness to make good on debt service. Since the borrower's option to default is essentially a put option on the underlying asset (a la Merton (1974)), we may think of the default barrier in equation (2) as analogous to the early exercise boundary for a put option that captures his default decision.

The composite function $\gamma(1-\theta)$ is inversely proportional to the propensity to default. It also accounts for transactions costs and other frictions in the loan market that make default less attractive to the borrower. For example, if default were costless (i.e., if there were no credit repercussions, no restrictions on refinancing, and no loan fees) and borrowers could move seamlessly to new assets, then borrowers would default the moment $H < L$; i.e., γ would equal 0. Therefore, the parameter γ encapsulates exogenous loan market frictions, recourse laws, etc., that modulate a borrower's willingness to pay in addition to endogenous borrower characteristics that drive default behavior. It is feasible to extract γ using a data set of defaulted bonds where the default level is known from empirical history, or by fitting an implied γ to match a distressed bond's current price. Note that γ embeds equilibrium behavior about default propensity that is based on past default occurrences in the economy as well as strategic behavior by the borrower.

Restructuring approaches: We consider debt that may be restructured by (i) writing down the outstanding amount L, (ii) reducing the interest rate c on the loan, or (iii) swapping debt for an equity stake, i.e., a fractional share $\theta \in (0,1)$ in the assets of the borrower. For example, with mortgages, the equity stake is obtained through a partial ownership in the home's value above a threshold; and for sovereign debt, equity is granted through warrants on government assets or additional guarantees.

Components of the restructured loan value with default risk: Define $\tau < T$ as the horizon for the restructured debt portfolio. Without loss of generality, we normalize the initial value of assets to $H_0 = 1$, and note that there are three components to consider in valuing loans to the horizon τ: (a) the value under default, which occurs when $H_t \leq D, t < \tau$, triggered when the asset price touches the barrier, and involves a first-passage probability under the risk-neutral measure; (b) the value under solvency (i.e., $H_t > D, \forall t \leq \tau$), where the loan does not default until τ; and (c) the value of the lender's equity stake in H when the equity sharing option is in-the-money (i.e., $H_t > D, \forall t < \tau,$ and $H_\tau > K$).

Binomial model: We implement this continuous-time model on a discrete-time binomial tree, allowing for coupons on debt. The underlying stochastic process for H_t in equation (1) is modeled in discrete time over a constant time step h. A standard binomial approach based on Cox, Ross, and Rubinstein (1979) is used where at each time t, the value process branches to two values at time $(t + h)$:

$$H_{t+h} = H_t \exp[\pm\sigma\sqrt{h}]$$

Since the pricing of the loan and underlying default and appreciation sharing features are to be priced under the risk-neutral measure, the probability of the up branch is given by $q = \frac{R-d}{u-d}$, where $R = \exp[rh]$, $u = \exp[+\sigma\sqrt{h}]$, and $d = \exp[-\sigma\sqrt{h}]$. The probability of a down move is $(1 - q)$. If debt has maturity T, then we build a tree out for $n = T/h$ periods; e.g., if $h = 1/4$ (one quarter) and the debt horizon is $T = 5$ years, then the model has $n = 20$ periods.

Debt is assumed to have a coupon per annum of rate c continuously compounded. Hence the per period coupon will be $C = L\left(e^{ch} - 1\right)$. This coupon will be added to the cash flows of debt, and the entire stream of cash flows will be discounted on the binomial tree via backward recursion to get the current price of the security. Backward recursion is undertaken as follows. At time T, the cash flows of debt will be $L + C$ for all levels of H_T, except where $H_T \leq D$ in which case debt is worth ϕH_T, or if $H_T > K$, then the value is $L+C+\theta(H_T-K)$. To arrive at the expected present value (price) of debt, these terminal cash flows are then discounted back on the binomial tree, weighted by probabilities $\{q, 1 - q\}$, adding in the coupon cash flows period by period, until we reach time zero. At any point $t < T$ in time, if $H_t \leq D$ then default is triggered and debt value is set to ϕH_t.

Our model includes an extension to shared appreciation features for the lender/investor, which are not considered in earlier work, and are easily dealt with in our tree model. Whereas earlier work examines the effects of leverage in the presence of positive equity, i.e., debt is less than collateral value, here we specifically examine the converse, where collateral has lost value and is less than debt, making it distressed. Our focus is implementation oriented in that we are interested in constructing optimal portfolios with much shorter horizons than the underlying debt, where we consider higher-order moments in the restructuring and portfolio construction process.

3 Risk and Return Analysis

This tree-based solution is simple to implement and the value of the loan as a function of the loan parameters $\{L, K, \theta, c\}$ is easily computed. We focus on modifications that comprise principal reductions, i.e., modulation of LTV (the loan to value ratio L/V) by reducing L, and to a lesser extent, modifications using shared appreciation and rate modifications, i.e., modulating $\{\theta, K, c\}$.

(a) Resetting LTV delivers gains: Consider an underwater loan with outstanding principal $L = 1.2$ where $H_0 = 1$. We observe from Figure 1 that the loan value initially increases with LTV, hitting its peak at roughly $LTV = 1.137059$ (with an optimal loan value of 1.2081) when the willingness-to-pay parameter is $\gamma = 0.20$, after which point the loan value drops precipitously and tapers off at $\phi H = 0.9$ (i.e., sufficiently high LTV triggers immediate default). Setting LTV too low results in loss in value from giving away too much principal, and is almost linear in effect. On the other hand, setting LTV too high leaves the borrower with too much debt relative to underlying collateral value, making the probability of default high, and entails loss from expected bankruptcy costs that reduce the value of the loan. Hence, a "sweet" optimal spot between these two trade-offs is detectable with our pricing model. The prescription in this example is a principal write-down from $L = 1.20$ to $L = 1.14$. The graph also shows that the slope of loan value to the right of the optimal point is much steeper than the slope on the left. This suggests erring on the side of restructuring debt to

a lower LTV than a higher one.

(b) Willingness-to-pay matters: For comparison, in Figure 1 we plot side-by-side the shapes of the loan value function for willingness-to-pay $\gamma = 1.20$ (as in (a) above) and $\gamma = 0.05$. We make similar observations when willingness to pay is substantially lower ($\gamma = 0.05$), but naturally, the function hits its peak at a much lower LTV of 0.987469 (with an optimal loan value of 1.0539). Hence, when willingness to pay is low, a much greater write-down is required in order to maximize loan value.

We also examine the shape of the loan value function for different levels of H (shown in Figure 2) as well as for willingness-to-pay coefficients γ (shown in Figure 3). In the former, we observe that for a loan with an initial principal of $L = 1.02$, the loan value steadily increases with H, with a sharp increase in value beginning at roughly $H = 0.8$ and reaching its maximum value at roughly $H = 1.0$. After the cusp point of $H = 1.0$, the loan turns sufficiently into positive equity range, and the option to default sharply drops off in value, with a corresponding increase in loan value. In contrast, the value of a loan with an initial loan principal of $L = 1.20$ does not reach its maximum value until the underlying asset value is roughly $H = 1.15$.

In Figure 3, we observe that the value of a loan with an LTV of 1.02 steadily increases with γ at first, then is a fairly constant function of γ (maintaining a loan value of roughly 1.117 whether γ is 0.20 or 0.50), suggesting that after a certain point, the borrower's propensity to default does not change no matter how willing/unwilling he is to pay. On the other hand, the value of a loan with an LTV of 1.20 remains constant at 0.9 until the willingness-to-pay coefficient approaches $\gamma = 0.20$, suggesting that such an underwater loan encourages strategic defaulting for borrowers who do not naturally have a high will to pay.

Having seen how the value of distressed debt changes with various parameters, we now turn to investor returns from investing in this asset class.

3.1 Returns over the holding period

A current or potential investor in distressed debt is interested in improving the return distribution over the portfolio horizon τ. We calculate the distribution of returns on unrestructured and restructured debt of maturity T over the chosen horizon τ. Specifically, we may calculate the distribution of $P_\tau(H_\tau)$ (the loan value at time $t = \tau$), which, following from equation (1), depends on the distribution of $H_\tau = H_0 \; e^{(\mu - \frac{1}{2}\sigma^2)\tau + \sigma\sqrt{\tau}Z}$, where $Z \sim N(0,1)$. The preceding equation is the solution to the stochastic differential equation (1). Consequently, continuous holding period returns are a random variable $R_{0,\tau} = \ln[P_\tau(H_\tau)/P_0]$, induced by the distribution of H_τ.

However, we are not interested only in the unconditional distribution of $R_{0,\tau}(H_\tau)$ at time τ, because in the interval from time $t = 0$ to $t = \tau$, the loan can default at any time up to horizon τ if the value of the underlying H_t drops to the default barrier D. Therefore, the returns from holding debt from time $t = 0$ to $t = \tau$ are weighted by the conditional probability of reaching H_τ given that no default occurs in between. Also, when default has not occurred, then the returns must include accumulated interest that is collected from the loan over time interval $[0, \tau]$. We assume that no accrued interest is collected when the loan defaults, and that the recovery amount is received at the horizon τ. The returns are thus as follows:

$$R_{0,\tau} = \begin{cases} \ln\left[\frac{1}{P_0}\; P_\tau(H_\tau)\right] + \mathcal{I}_{0,\tau} & \text{if } H_t > D, \text{ for all } t \in [0,\tau] \\ \\ \ln\left[\frac{1}{P_0}\; \phi\; \min(D, H_0)\right] & \text{if } H_t \leq D, \text{ for any } t \in [0,\tau] \end{cases} \tag{3}$$

where $\mathcal{I}_{0,\tau}$ is the accumulated interest compounded to time τ at the risk free rate over n_τ periods. The first outcome is when no default occurs and the second is for the default case. The accumulated interest is computed as the discounted stream of cashflows.

$$\begin{aligned} \mathcal{I}_{0,\tau} &= C\left[1 + e^{r_f h} + e^{2r_f h} + e^{3r_f h} + \ldots + e^{(n_\tau - 1)r_f h}\right] \\ &= \frac{C \cdot (\exp[r_f\; h\; n_\tau] - 1)}{\exp[r_f\; h] - 1} \end{aligned} \tag{4}$$

where $h = T/n$ and $n_\tau = \tau/h$, and as before, $C = L\left(e^{ch} - 1\right)$.

In order to compute the moments of this return we need the following probability function for the density of H_τ conditional on no default $(H_t > D, \forall t \leq \tau)$, and this is easily derived from standard barrier option mathematics, see for example Derman and Kani (1997).

$$
\begin{aligned}
\text{Prob}[H_\tau | H_0, H_t > D, 0 \le t \le \tau] &= A_1 - \left(\frac{D}{H_0}\right)^{\frac{2\mu}{\sigma^2}-1} \cdot A_2 \\
A_1 &= n\left\{\frac{\ln(H_\tau/H_0) - (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}\right\} \\
A_2 &= n\left\{\frac{\ln(H_\tau \cdot H_0/D^2) - (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}\right\}
\end{aligned}
\tag{5}
$$

where $n\{\cdot\}$ is the normal probability density function. Therefore we weight the first component of returns in equation (3) with $\text{Prob}[H_\tau | H_t > D, 0 \le t \le \tau]$ given above, and the second component of those returns with $\text{Prob}[H_\tau | H_0, H_t \le D, 0 \le t \le \tau] \equiv \text{Prob}[H_\tau | H_0] - \text{Prob}[H_\tau | H_0, H_t > D, 0 \le t \le \tau]$, which gives the overall probability of default over horizon τ when integrated over H_τ-space.

We can then compare the default probabilities and return distributions for varying levels of initial LTV. Table 1 provides a numerical comparison of one-year loan return distributions and probabilities of default under the various parameter levels. We observe that the expectation and uncertainty of returns generally increase with LTV and decrease with γ until a certain threshold, after which point the default scenario dominates other outcomes. When the willingness-to-pay parameter is too low, and the LTV and consequent negative equity are too high, then the loan ends up in automatic default (i.e., forced default), as may be seen from Panel B of Table 1.

We also observe that the expectation and uncertainty of returns decrease with the default recovery rate ϕ. At a higher recovery rate, loans before restructuring are priced higher and hence the returns are lower. Thus, yield pick ups from restructuring are naturally much greater when the loss on bankruptcy $(1 - \phi)$ is higher (30% versus 10%). For illustrative purposes, in most of the ensuing analyses, we proceed to examine the risks and returns under the more conservative default recovery rate of $\phi = 0.9$.

4 Optimal Restructuring

To set ideas and establish a framework, we examine the restructuring process for a single loan. (i) We first determine the fair value of the loan before restructuring, and assume that this is its base price P_0. The default probability and return distribution of this loan P_τ, at investment horizon τ depends on the return distribution of the underlying asset H_τ, after properly accounting for intermediate default. (ii) Likewise, we also compute the default probability and return distribution of the restructured loan, and compare this against that of the unrestructured loan to assess the benefits of restructuring. We compare the following cases.

1. The loan return distribution and expected utility from investing in the loan for a horizon τ, *before restructuring*.
2. The loan return distribution *after restructuring*, i.e., from writing down the debt principal to a level where the LTV is such that the *loan value is optimized*.
3. Returns from changing the LTV such that the *certainty equivalent* (moment-based risk-adjusted gain) of the restructured loan relative to that of the original loan is maximized.
4. Returns from reducing the interest rate on the loan in an amount equal in relief offered through the principal reduction as calculated in case (2); i.e., offer a "service-equivalent" restructuring to the borrower.

Minimally underwater loans: Table 2 shows an example of the first three cases and provides a numerical comparison of one-year loan return distributions, before and after restructuring, for a single loan that is mildly under-collateralized (i.e., its LTV is 1.02) and where the recovery rate on default is $\phi = 0.9$ and ability to pay is not an issue (we later explore case 4, rate reduction, when we introduce uncertainty in a borrower's ability to pay). The other parameters of the loan are shown in Table 2.

Prior to restructuring (case 1), a loan with an LTV of 1.02 is valued at \$111.65 (per hundred dollars) and has a 0.00% probability of default (over the next year) when the willingness-to-pay default risk parameter is $\gamma = 0.20$ (Panel A). The mean return on the loan is 2.03% with a standard deviation of return of 0.10%; the distribution is negatively skewed (−13.1706) and fat-tailed (kurtosis of returns is 311.0446). Here, we observe that the value-optimizing $LTV = 1.137059$ is greater than 1.02; i.e., reducing LTV will drop loan value with such a high willingness-to-pay (γ). Hence, no modification through principal reduction is necessary. Thus, the remaining modification cases in the table in Panel A are left blank.

In contrast, when the willingness-to-pay is low, i.e., $\gamma = 0.05$ (Panel B), we observe greater opportunity for and substantially greater gains from loan restructuring through principal write downs. Prior to restructuring, a loan with an LTV of 1.02 is valued at $1.0307 and has a 17.55% probability of default (over the next year) when the willingness-to-pay default risk parameter is $\gamma = 0.05$. The mean return on the loan is 3.37% with a standard deviation of return of 9.53%; there is mild negative skewness (−1.4875) and mild kurtosis (3.4568, in excess of the normal level of 3).

Post-restructuring, the LTV is set to a level that maximizes the loan's value, i.e., 0.987469 (Panel B, case 2a), restoring the loan to positive equity. The maximized price of the loan is $1.0539, bringing down the holding-period probability of default to 1.84%; the mean return over the holding period increases to 5.05%, i.e., a gain of 1.68% and the standard deviation falls dramatically to 3.91% (from 9.53%) because the default outcomes are drastically curtailed. However, there is some increase in negative skewness (to −5.0453) and appreciable increase in kurtosis to 31.0385, which is, in part, on account of the much lower standard deviation. We also consider the case where the lender takes an upside share in the restructuring, through a shared-equity-appreciation loan (SEAL). If we perform a value-maximizing restructuring but now allow for shared appreciation with strike $K = 1$ and $\theta = 0.2$ (case 2b), the new return distribution on this loan has a mean of 6.30% with a standard deviation of 4.30%.

Deeply underwater loans: Similarly, in Table 3 we consider a loan that is deeply underwater with $LTV = 1.2$, where the recovery rate on default is still $\phi = 0.9$ and ability to pay is not an issue. When the willingness-to-pay default risk parameter is $\gamma = 0.20$ (Panel A), this loan is valued at $1.0548, and has a one-year default probability of 37.39% and expected return of 5.65% (with a standard deviation of 18.35%).

If we restructure this loan with a principle write-down such that $LTV = 1.137059$ (case 2a), probability of default drops to 0.91% and the new return distribution on this loan has a mean of 16.53% with a standard deviation of 4.67% (Figure 4). If we perform a value-maximizing restructuring but now allow for shared appreciation with strike $K = 1$ and $\theta = 0.2$ (case 2b), the new return distribution on this loan has a mean of 14.32% with a standard deviation of 3.60%. We observe that skewness is less negative and kurtosis declines with an increase in the SEAL share θ. Thus, at higher levels of θ, the restructured debt return distribution is more symmetric, though kurtosis levels remain high. The return distributions for this loan under the various restructuring scenarios are shown graphically in Figure 4. Consistent with the distribution parameters presented in Table 3, we observe that with restructuring, the left tail of the return distribution becomes less pronounced. And for a shared-appreciation loan, the left tail is even further reduced. This is a vast improvement in returns over the unrestructured loan, especially when collateral is deeply impaired.

When the willingness-to-pay default risk parameter is substantially lower with $\gamma = 0.05$ (Panel B), we again observe greater gains to restructuring. Prior to restructuring, a loan with an LTV of 1.20 is valued at \$0.90, since the low willingness-to-pay triggers automatic default when the loan is deeply underwater. If we restructure this loan with a principle write-down such that $LTV = 0.987469$, the default probability drops to 1.84% and the new return distribution on this loan has a mean of 18.61% with a standard deviation of 3.91%. If we perform a value-maximizing restructuring but now allow for shared appreciation with strike $K = 1$ and $\theta = 0.2$, the new return distribution on this loan has a mean of 19.86% with a standard deviation of 4.30%. We again observe that as the SEAL share θ increases, skewness is less negative and kurtosis declines, but now the expected return also increases. However, increasing θ does not necessarily result in a dominating restructured security because the standard deviation of returns may also increase.

Note that when comparing restructured loan distributions, the higher moments are the same whether the original loan was at $LTV = 1.02$ or $LTV = 1.20$; only the expected return is different. That is, if we restructure a loan using the same principal write down and shared appreciation parameters, the shape of the distribution itself is the same regardless of how far underwater the original loan was; there is simply a shift in the mean return of the distribution depending on the original LTV level, because the original fair price varies with the LTV at inception. In this sense, accounting for all moments of the return distribution, the restructuring is optimized both by negotiating a good price at which the distressed loan is bought by the investor, as well as the implementation of an optimal restructuring taking into account all higher-order moments. We turn to this optimization next (case 3), undertaking it in a utility framework so as to capture the effect of all moments of the return distribution of loans, both before and after restructuring.

4.1 Higher-order Moments

In order to compare the gains for an investor that cares about higher moments beyond mean and variance, we examine the case of a CRRA investor with risk aversion coefficient $\beta = 3$ (we also footnote results using $\beta = 5$ for comparison). We assume a CRRA utility function before and after restructuring based on wealth $W = 1 + R$, where R is return:

$$U(W) = W^{1-\beta}/(1-\beta)$$

The last column in Table 2 shows that the utility gains from restructuring are substantial, and at the optimal LTV, the CRRA utility is -0.4828 versus a utility of -0.4556 prior to the restructuring of a loan with an original $LTV = 1.02$ when the willingness to pay risk parameter is $\gamma = 0.05$. To convert the improvement in utility from debt restructuring into basis points, we calculate the certainty equivalent (CE, in basis points) between the return distributions of the base case loan and the restructured one using the CRRA utility function. The expression for CE is based on the following equivalence that requires that the base case wealth be increased by a factor of $(1+CE)$ to make the unrestructured loan equal in expected utility to that from the restructured loan. The equivalence is:

$$\sum_i p_i((1+R_i^b)(1+CE))^{1-\beta}/(1-\beta) = \sum_i p_i(1+R_i^r)^{1-\beta}/(1-\beta)$$

Note that we use R_i^r to denote returns of the restructured loan and R_i^b for the loan before restructuring. The probability of seeing outcome R_i in state i is denoted p_i. Also note that the RHS is just the expected utility of the restructured loan, and may be written as $E[U(W^r)]$. The LHS may be written as $(1 + CE)^{1-\beta} \times \sum_i p_i(1 + R_i^b)^{1-\beta}/(1 - \beta)$ where$E[U(W^b)] = \sum_i p_i(1 + R_i^b)^{1-\beta}/(1 - \beta)$ is the utility of the loan before restructuring. Hence we may solve for CE to be:

$$\text{Certainty Equivalent} \equiv CE = \left[\frac{E[U(W^r)]}{E[U(W^b)]}\right]^{1/(1-\beta)} - 1$$

We present the results of this computation in basis points in the tables.

Minimally underwater loans: Applying this computation to compare the base case loan and the restructured one from Table 2, we find that $CE = 0.0294$, i.e., a material 294 basis points pick up from restructuring (case 2a of Panel B, 399 bps for risk aversion coefficient $\beta = 5$). Applying this same computation but now allowing for shared appreciation, we observe even greater gains in CE. When the SEAL share is $\theta = 0.2$, the yield pick up is higher than in the case without the shared appreciation, and is 411 basis points (case 2b of Panel B, 513 bps for $\beta = 5$). As risk aversion increases, restructuring dials down risk, making risk-adjusted returns more attractive.

The third case we consider is where we maximize the CE gains from restructuring. In this case, instead of choosing the LTV to maximize loan value, we choose LTV to maximize CE. It turns out that the CE-maximizing LTV is 0.985308 (slightly higher than the loan maximizing LTV of 0.987469). The loan value and mean return at this LTV are slightly lower, but the standard deviation is also lower at 3.61% (versus 3.91%). However, both skewness and kurtosis are slightly higher, the former is more negative and the latter is larger, because when maximizing utility, the optimization results in portfolios that are less penalized for skewness and kurtosis. The certainty equivalent (relative to case 1) is $CE = 0.0297$, i.e., 297 basis points (405 bps for $\beta = 5$), which implies further increases in utility gains over case 2a of three basis points.

Deeply underwater loans: In Table 3, we undertake the same analysis for a loan with an LTV=1.20, i.e., a loan that has much more negative equity and more default risk, and thus a lower price, one that can be acquired at a steeper discount. The main difference in results is that the CE basis points yield pick up is much larger, around 1,500 basis points (at $\beta = 5$, these pickups increase by 200-400 bps for the increased risk aversion). But when adjusted for the magnitude of principal forgiveness, we observe that the CE gains per dollar writedown are greater when restructuring less distressed loans than when restructuring such deeply underwater loans; that is, the CE pickup to principal writedown ratio is greater in the former (Table 2, Panel B) than in the latter (Table 3, Panel B).

In the case where willingness to pay is very low, as in Table 3 (Panel B), the change in CE pickup declines when $\beta = 5$ instead of $\beta = 3$. These loans were almost certain to default with return distributions of low dispersion, and restructuring in fact increases dispersion, so as we go from $\beta = 3$ to $\beta = 5$, the CE pickup declines marginally.

The Magnitude of Yield Pick Ups: Why are these yield pick-ups reasonably large? In Panel A of Table 3, where the loan balance is 1.20 relative to an underlying asset value of 1.00, the fair value of the loan is \$1.05 and the probability of default is 37.39%. Since the loan has a high probability of default and recovers only a fraction, $\phi = 0.9$, of the collateral value on default, the expected loss is high. We see that the default barrier, the collateral value at which default occurs, is equal to $D = Le^{-\gamma} = 1.20e^{-0.2} = 0.982477$, of which only 90% (i.e., \$0.88) is recovered. This amount recovered is \$0.32 less than the loan balance. After restructuring, the probability of default falls to less than 1%, and hence, most of the expected loss is mitigated, which after risk adjustment through the utility function, leads to a yield pick up of 15.60%, a fair fraction of the averted loss. High returns are not unusual in distressed debt investing given the magnitude and multitude of risks involved, see Gilson (1995), Figure 1 for example, who states that "it is not uncommon for investors in distressed claims to seek annualized returns in the range of 25–35%" (page 19), a number that is of comparable magnitude to the ones we see in our simulations. Our paper offers a parsimonious framework for optimal restructuring that such investors may deploy.

4.2 Returns under Falling Collateral Values

Thus far, we have focused on the impact of loan restructuring under the assumption that collateral values are expected to appreciate, on average, by $\mu = 4\%$ annually. Although this assumption may hold generally, times in which principal write downs are most beneficial are precisely when collateral values over a one-year horizon are likely stagnating, or still declining. In Table 4, we examine the CE basis point pick ups on restructured loans under alternate collateral growth rates, $\mu = \{-4\%, 0\%\}$.

We observe that when collateral values are expected to stagnate or decline, the one-year expected returns (before restructuring) on underwater loans with an $LTV = 1.20$ are -14.48% and -6.31% for $\mu = -4\%$ and 0%, respectively, when the willingness-to-pay is $\gamma = 0.20$. When loan restructuring entails a principal writedown but no shared appreciation, we observe that the expected returns on these distressed loans now become positive, with post-restructuring expected returns of 8.90% and 13.02%, respectively. These distributional changes translate to yield pickups of 2,859 and 2,471 basis points, respectively, which quantifies, after risk-adjustment, the return gains from restructuring. This suggests that in the face of falling collateral values, such restructuring is greatly beneficial to current loan holders, but the distressed loans will be an attractive security to new investors only if they can negotiate a price at which the expected return of these securities is positive or above a threshold of risk-adjusted return that they require. Recall from Section 4 that after restructuring, the return distribution is always the same in higher-order moments, but that the mean return depends on the initial value or price paid for the unrestructured loan. Hence, our approach provides a framework for new investors to bid on distressed loans by setting reservation prices that will satisfy the needs of the risk-return tradeoffs in their existing portfolios.

When restructuring also entails shared appreciation of $\theta = 0.2$, we observe post-restructuring expected returns of 6.37% and 10.71%, which translates to certainty equivalent yield pickups of 2,528, and 2,220 basis points, respectively, in the range of expected returns required by investors as documented in Gilson (1995). Therefore, even when collateral value is expected to stagnate or mildly decline, restructuring delivers a product with low risk and low return that is attractive even for new investors, and certainly offers a large yield pick up for existing investors.

4.3 Portfolio Effects

The pooling of distressed loans enables further gains from diversification across risks emanating from variance, skewness, and kurtosis. We therefore extend our single loan framework to portfolios. In our previous analyses, we calculated the returns from holding a single loan over horizon τ using the barrier probability density function in equation (6), which allows for possible default in the intervening time. For portfolios, the joint density function for multiple loans, conditional on one or more defaults occurring prior to τ, is much more complicated, and is not tractable analytically or semi-analytically. Therefore, we simulate 100,000 draws of the joint outcome of collateral values at τ with a pre-specified pairwise asset correlation (i.e., either 0.70 and 0.95). As in the single-loan case, returns are generated from the same geometric Brownian motions with multivariate Wiener processes. We then use equation (6) to adjust the expected returns for each loan based on the probability of default prior to τ.

Portfolio Effects Before Restructuring: In Table 5, we compare the return distribution of a single loan and that of a two-loan portfolio using the same input parameters as before (in Table 2), with *no restructuring* (Panel A). Depending on the assumed correlation between the underlying assets securing the loans, we observe improvements in standard deviations and mild reductions in negative skewness. For an assumed asset correlation of 0.7, we observe a standard deviation of 6.60% (versus 9.53% for a single-loan portfolio) and a skewness of 4.0050 in portfolio returns, which translates to a gain in CE of 87 basis points purely from diversifying into an additional unrestructured loan; for an asset correlation of 0.95, we observe a gain in CE of 71 basis points. As expected the gains from diversification are lower when the correlation of asset values is high. We choose high correlation levels as these are expected in economic downturns.

We also examine the return distribution of a hundred-loan portfolio, and we observe further CE gains from this increased diversification. For an asset correlation of 0.70, the hundred-loan portfolio gains 102 basis points over the single-loan portfolio, and 15 basis points over the two-loan portfolio. Most of these gains come from a reduction in the standard deviation of returns of the portfolio, though there is also some reduction in skewness and kurtosis.

Portfolio Effects After Restructuring: In comparing the gains in CE *with loan restructuring* (Panel B), we observe that the gains from diversification of already restructured loans are less pronounced; for an asset correlation of 0.7, we observe a CE gain of 22 basis points above and beyond the CE gain realized by the single, restructured loan, and for an asset correlation of 0.95, we observe a relative CE gain of 20 basis points. The gains from diversification are not very large when we go from the two-loan portfolio to the hundred-loan portfolio.

As shown in Panel C, further gains accrue when shared appreciation is implemented in the restructuring. With restructured shared equity appreciation loans (SEALs, Panel C), diversification substantially reduces the standard deviation, skewness, and kurtosis of returns when going from a single loan to a diversified portfolio, resulting in a CE yield pick up. But again, the gains come predominantly from restructuring itself and not from diversification. For an asset correlation of 0.7, we observe a CE gain of 27 basis points above and beyond the CE gain realized by the single, restructured loan, and for an asset correlation of 0.95, we observe a relative CE gain of 23 basis points.

Thus, we see that most of the CE gains come from restructuring individual loans. Overall, after restructuring, the gains from diversification are not significant (partly on account of high correlation levels), whereas portfolios of unrestructured loans benefit more from pooling to achieve diversification. Still, when restructured loans are combined into a portfolio, there is a material reduction in standard deviation, skewness, and kurtosis risks, particularly when the correlation across loans is lower. Hence there is some gain in CE when going from a single-loan portfolio to a diversified portfolio.

4.4 Ability-to-Pay Risk

Thus far, we have abstracted away from a borrower's *ability* to pay, focusing only on differing levels of a borrower's *willingness* to pay (i.e., γ). Recall that the loan value under willingness-to-pay risk was denoted P. We now explore the risks and returns to debt investors when we introduce ability-to-pay uncertainty. To account for this alternative source of default risk, the loan value with ability-to-pay (ATP) risk is now calculated as:

$$P_{ATP} = P \cdot \left[1 - N\left(\frac{c \cdot L - \mu_I}{\sigma_I}\right)\right] + \phi \cdot H \cdot N\left(\frac{c \cdot L - \mu_I}{\sigma_I}\right), \qquad (6)$$

where P represents the value of the loan when the borrower's willingness to pay is an issue but his ability to pay is not, μ_I represents the borrower's expected annual (normalized to unity) debt service revenue, and σ_I represents revenue uncertainty. Assuming normally distributed income, the probability of an ATP-based default is $N\left(\frac{c \cdot L - \mu_I}{\sigma_I}\right)$, where $N(\cdot)$ is the cumulative normal distribution function. Hence, the original loan value is multiplied by the probability that there is no ATP default, plus the recovery value weighted by the probability of an ATP default.[14]

Principal Reduction offers effective restructuring even with ability-to-pay risk: Restructuring by writing down the loan principal L now affects loan pricing not only by lowering the default barrier D, making it less likely to trigger willingness-to-pay default (see equation (2)), but also by entering the function for the probability of ability-to-pay default (see equation (6)), making it less likely as well. In addition, as the coupon-rate c changes, the value-maximizing LTV will also change. Hence, both principal reduction and rate relief interact in the optimized restructuring.

In Table 6, we examine the return distributions under various levels of willingness to pay ($\gamma = \{0.20, 0.05\}$) when the borrower's expected income is $\mu_I = 0.12$, i.e., three times the coupon rate of 0.04, which corresponds to a loan service to income ratio[15] of 0.33 with income uncertainty $\sigma_I = 0.06$.

We observe that when the willingness-to-pay parameter is $\gamma = 0.20$ (Table 6, Panel A), a loan with an original LTV of 1.20 and coupon rate of 0.04 is priced at 1.0370, with an expected return and standard deviation of 5.47% and 16.90%, and a default probability of 44.59% over a one-year horizon (case 1). In comparison this same loan was priced at 1.0548 with a default probability of 37.39% when there was no ATP risk, as shown in Table 1, Panel A. Therefore, the reduction in loan value from ATP risk over and above that from willingness-to-pay risk can be substantial, in this case resulting in a haircut of 178 basis points. If we write down the loan principal such that the value of the loan is maximized (case 2), the default probability decreases to 11.52%, and the new return distribution on this loan has a mean of 15.55% with a standard deviation of 4.48%, which translates to a CE yield pickup of 1,395 basis points.

Rate reduction is not optimal when there is willingness to pay risk: In practice, lenders have been reluctant to make principal reductions in loan renegotiations (Asquith, Gertner, and Scharfstein (1994); Ghent (2011)), but have been willing to provide rate relief. We examine these two alternatives here. If we keep the loan principal fixed and, instead, perform

[14]This model for ability to pay risk was introduced in Das (2012).

[15]In the case of mortgages, the HAMP policy document advocates an HTI (home loan service to income) ratio to support lending of 0.31–0.38.

a coupon writedown such that the annual loan service payment is the same as in case 2, the new return distribution has a mean of 5.09% with a standard deviation of 16.61%, which translates to a CE yield pickup of −23 basis points, i.e., risk-adjusted for higher moments, the investor is worse off after the restructuring. Thus we see that when restructuring distressed debt, the source of relief to borrowers has very different value implications for investors, after holding the total debt service burden on borrowers the same. In fact, here, loan rate reductions are *not optimal* approaches for restructuring of loans (i.e., a value-improving coupon-rate reduction does not exist under the given parameters).

Similar observations apply when we explore the yield pickups from principal versus coupon writedowns under a willingness-to-pay default risk parameter of $\gamma = 0.05$ (Panels B1 and B2). We first consider a loan that is deeply underwater with $LTV = 1.20$ (Panel B1). In this case, a rate reduction is much worse, and is not effective at all since the borrower defaults with certainty, whereas the principal reduction delivers a higher CE gain of 1,714 basis points. For a loan that is mildly underwater with $LTV = 1.02$ (Panel B2), the loan value is 1.0185, a haircut of 122 basis points in price when compared to the same loan when there was no ATP risk, which was priced at 1.0307 (Table 1, Panel B). Since negative equity is small and the loan is not as deeply distressed (with a pre-restructuring default probability of 25.25%), the yield pick up from restructuring is lower, amounting to 268 basis points.

Rate reduction is optimal when there is no willingness to pay risk: In Table 7, we isolate the ability-to-pay issue, abstracting away from the borrower's willingness-to-pay. That is, we set the willingness-to-pay default risk parameter to $\gamma = 100$ so that the default barrier approaches zero (i.e., a borrower is always willing to pay, even as the underlying asset declines in value). Here, we observe substantial CE gains to applying coupon-rate reductions. For instance, when the borrower has an expected income of $\mu_i = 0.02$ with $\sigma_i = 0.02$ (Panel A), a loan with $LTV = 1.20$ and coupon rate of $c = 0.04$ is priced at 0.9335 (Case 1). Restructuring the loan such that a value-maximizing principal writedown is undertaken (Case 2), yields slight improvements in loan price, holding-period returns, and likelihood of default, with an ultimate CE gain of 13 basis points. However, a coupon writedown such that the annual payment is the same as in the case of the principal writedown (Case 3), yields even greater improvements, with an ultimate CE gain of 100 basis points. Performing a value-maximizing coupon-rate reduction (Case 4) yields the greatest improvements to the investor, with an ultimate CE gain of 1,145 basis points. The value-maximizing reduction in coupon is large enough to be almost coupon forbearance. Thus, we see that coupon-rate reductions add the greatest value when willingness-to-pay is not a concern but ability-to-pay is a serious issue.

4.5 Pareto Optimal Gains from Restructuring

Restructuring benefits the borrower (corporation, home owner, or sovereign) through loan relief, either in terms of a reduced interest rate or principal write down, resulting in a lower service burden. Restructuring also benefits the owner of the loan (lender or investor) by eliminating the deadweight costs of bankruptcy, and by mitigating the debt overhang problem. When the investor buys the loan from the original lender, the price negotiated for the transaction determines how the gain from restructuring is shared between both parties.

The benefits to the owners of the loan are expressed in terms of the restructuring yield pick up in preceding sections. This yield pick up, ranging approximately from 1500 to 2800 basis points, *does not* include gains to the borrower. Therefore, the aggregate benefits of restructuring are understated in our analysis. This makes the case for imposition of eminent domain even more compelling. Additionally, by mitigating possible contagious default across borrowers, systemic risk is reduced, and societal value is preserved. These gains are also not factored in. This massively Pareto optimal solution, often impeded by inefficient distressed debt markets, calls for regulatory action or redesign of debt markets.

5 Concluding Discussion

This paper discusses how to restructure a portfolio of distressed debt, what the gains are from doing so, and attributes these gains to restructuring and portfolio effects. We develop a model for the pricing and optimal restructuring of distressed debt portfolios, i.e., loans that are underwater and at risk of borrower default, where willingness to pay and ability to pay are at issue. Debt restructuring involves optimization where the investor has control over the return distribution of distressed debt via restructuring. It also requires optimization over all moments, not just mean and variance. Even under moderate deadweight costs of bankruptcy, restructured debt return distributions are very attractive to fixed-income investors, with yield pick-ups in the hundreds of basis points. Furthermore, the shared appreciation feature matches investors' skewness preference. Diversification of restructured loans results in additional, though modest, gains in certainty equivalent basis points. The approach is general enough to be broadly applicable, applying to distressed sovereign, corporate, and mortgage debt. The general use of eminent domain to clear the securitization induced restructuring logjam and effect market-wide change is feasible because the gains from restructuring are huge, and result in a Pareto optimal outcome for all parties concerned.

The source of these restructuring gains can be elucidated and modulated in the model, and derives from careful management of the default put option held by the borrower, and to a lesser extent, from diversification of the higher-order risk moments. In addition, the model also exposes and mitigates the *incentive issues* that also affect loan value. Specifically, the lender's gains from restructuring are not simply due to staving off the deadweight costs of default; the fact that restructuring resuscitates hope of some gain for the borrower on the upside is just as crucial. Principal write-downs incentivize the borrower to maintain the value of the underlying collateral so that the lender stands to gain as well.

This reasoning provides the basis for advocating debt forgiveness in poorly performing countries in Europe in the current financial crisis. That is, "[w]hen a country's obligations exceed the amount it is likely to be able to pay, these obligations act like a high marginal tax rate on the country: if it succeeds in doing better than expected, the main benefits will accrue, not to the country, but to its creditors. This fact discourages the country from doing well at two levels. First, the government of a country will be less likely to be willing to take painful or politically unpalatable measures to improve economic performance if the benefits are likely to go to foreign creditors in any case. Second, the burden of the national debt will fall on domestic residents through taxation, and importantly through taxation of capital; so the overhang of debt acts as a deterrent to investment."(Krugman (1988)). In other words, mitigating the debt overhang problem (Myers (1977)) through principal restructuring is key.

In an environment of active de-leveraging, the gains from restructuring debt portfolios in an optimal manner, spread over the entire economy, are huge. The gains from these policy prescriptions are understated in our model because it does not account for the feedback effect of preventing default on supporting asset values in the case of home, corporate, or sovereign debt. In sum, active restructuring through the imposition of eminent domain reduces further losses from contagion and mitigates systemic default risk.

Figure 1:

Loan value for varying levels of LTV with no upside sharing by the lender and where ability-to-pay is not an issue. The input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.9$, volatility $\sigma = 0.04$, default risk parameter $\gamma = \{0.05, 0.20\}$, and coupon rate $c = 0.04$. We used $n = 100$ periods on the tree.

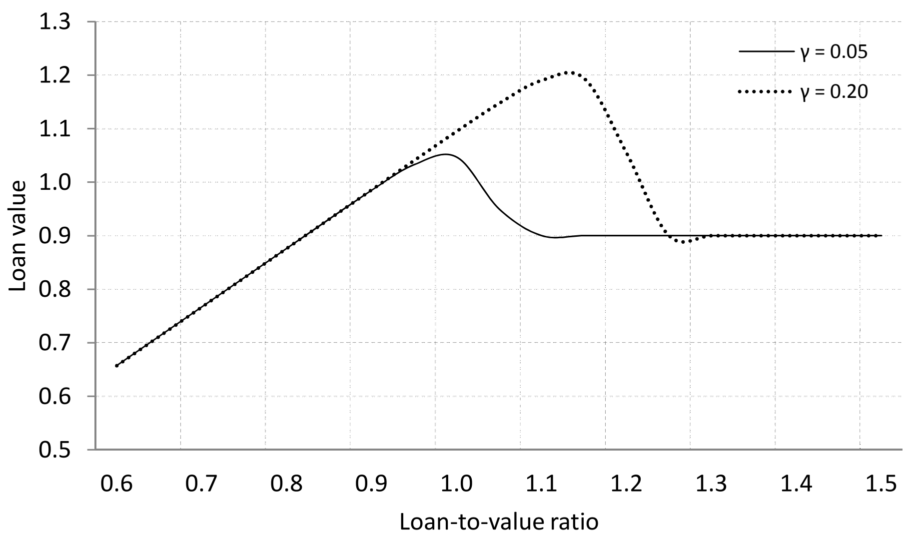

Figure 2:

Loan value for varying levels of H with no upside sharing by the lender and where ability-to-pay is not an issue. The input parameters are $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.9$, volatility $\sigma = 0.04$, default risk parameter $\gamma = 0.20$, and coupon rate $c = 0.04$. We used $n = 100$ periods on the tree. Note that when H is high and default risk is negligible the loan trades above par given the coupon rate is higher than the discount rate.

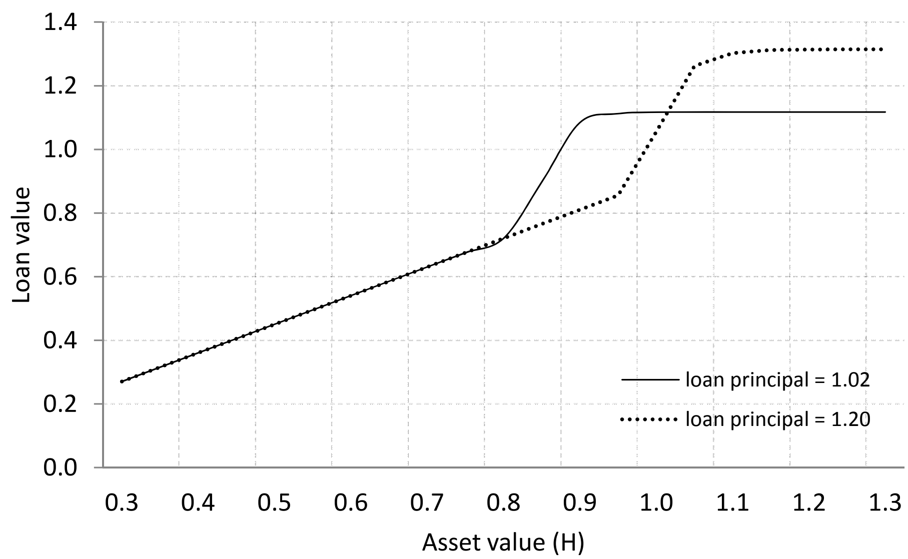

Figure 3:

Loan value for varying levels of γ with no upside sharing by the lender and where ability-to-pay is not an issue. The input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.9$, volatility $\sigma = 0.04$, and coupon rate $c = 0.04$. We used $n = 100$ periods on the tree. Note that when H is high and default risk is negligible the loan trades above par given the coupon rate is higher than the discount rate.

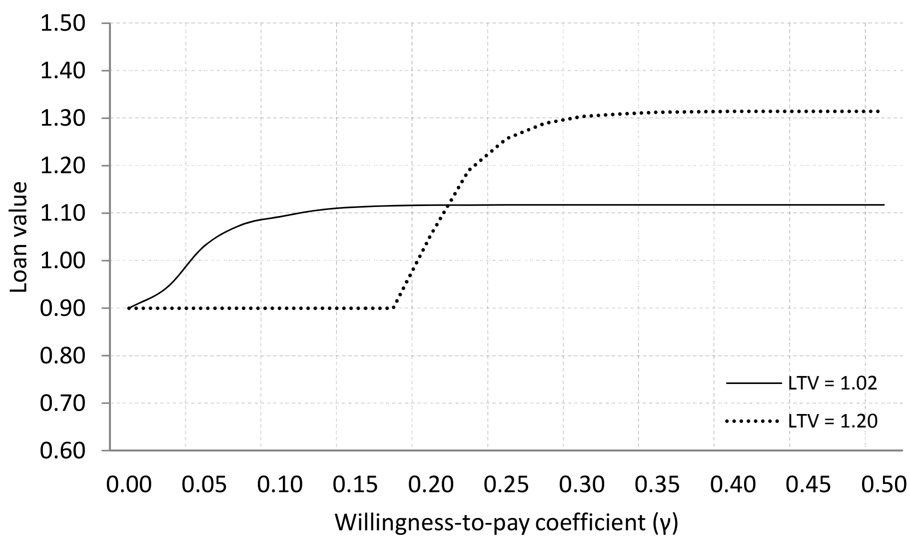

Figure 4:

Return distribution over a one-year holding period on a *restructured* loan with an initial LTV of 1.20 and purchase price of 1.0548, where ability to pay is not an issue and the willingness-to-pay default risk parameter is $\gamma = 0.20$. This graph compares the pre- and post-restructuring return distributions as outlined in Panel A of Table 3. Notice how the default tail shortens as we go from the original loan to the restructured one to the SEAL.

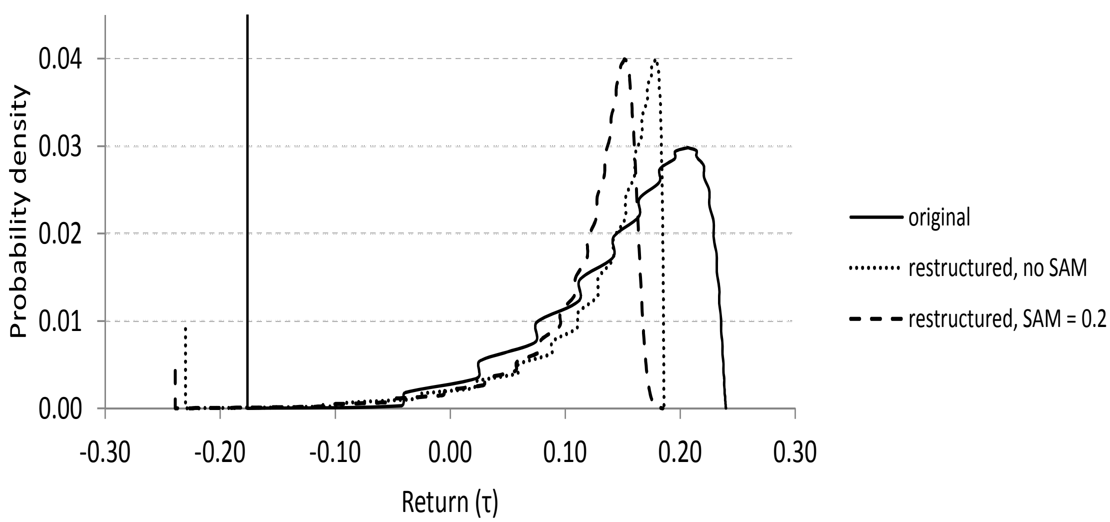

Table 1:

Probability of default, expected return, standard deviation, skewness, and kurtosis over a one-year holding period on an underwater loan with L=LTV=1.20 with no upside sharing by the lender and where ability-to-pay is not an issue. We compare return distributions under two different willingness-to-pay parameters: $\gamma = 0.20$ (Panel A), and $\gamma = 0.05$ (Panel B). The remaining input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = \{0.7, 0.9\}$, growth rate $\mu = 0.04$, volatility $\sigma = 0.04$, and coupon rate $c = 0.04$. We used $n = 100$ periods on the tree for loan pricing. "-D-" indicates that the loan will be in default from the outset and no return is earned.

LTV	Price	Pr(Def)	Mean	Stdev	Skew	Kurt
Panel A. Willingness-to-pay parameter $\gamma = 0.20$						
Under default recovery rate $\phi = 0.7$:						
1.02	1.1161	0.00%	0.0204	0.0016	-13.0226	304.8644
1.20	0.9331	37.39%	0.0699	0.2952	-0.4272	1.2703
Under default recovery rate $\phi = 0.9$:						
1.02	1.1165	0.00%	0.0203	0.0010	-13.1706	311.0446
1.20	1.0548	37.39%	0.0565	0.1835	-0.4217	1.2705
Panel B. Willingness-to-pay parameter $\gamma = 0.05$						
Under default recovery rate $\phi = 0.7$:						
1.02	0.9577	17.55%	0.0429	0.1850	-1.4784	3.4355
1.20	0.7000	100.00%	-D-	-D-	-D-	-D-
Under default recovery rate $\phi = 0.9$:						
1.02	1.0307	17.55%	0.0337	0.0953	-1.4875	3.4568
1.20	0.9000	100.00%	-D-	-D-	-D-	-D-

Table 2:

Optimization outcomes for a single loan with an original LTV of 1.02 and where the default recovery rate is $\phi = 0.9$ and ability-to-pay is not an issue. We optimize a loan where the initial home value is normalized to $H = 1$, and the loan balance is $L = 1.02$. Remaining loan maturity is $T = 5$ years, recovery rate $\phi = 0.9$, home price volatility is 4%, and the coupon rate on the loan is 4%. We compare optimization outcomes using two different willingness-to-pay parameters: $\gamma = 0.20$ (Panel A), and $\gamma = 0.05$ (Panel B). We assume a riskless rate of $r_f = 0.02$, and we use $n = 100$ periods on the tree for loan pricing. Returns on the loan are computed assuming an investment horizon of $\tau = 1$ year and a home price growth rate of $\mu = 0.04$. We calculate the returns and expected utilities from holding this loan in each of the following cases: (1) before restructuring; (2) *after restructuring*, i.e., after writing down the debt to a level where the LTV is such that the loan value is optimized. (3) after writing down the debt to a level such that the certainty equivalent of the restructured investment relative to that of the original loan is maximized. When restructuring entails shared appreciation, we set the strike $K = 1$. The overall probability of default as well as mean, standard deviation, skewness, and kurtosis of the return distribution with horizon τ are provided below. The certainty equivalents (CE) are computed for $\beta = 3$ (in a CRRA utility function, $U(W) = W^{1-\beta}/(1-\beta), W = 1 + R$) relative to the base case loan before restructuring. "—" indicates that reduction in principal does not lead to an improvement in the loan value, and hence no restructuring is undertaken.

Case	LTV	Price	Pr(Def)	Mean	Stdev	Skew	Kurt	E[U(W)]	CE
			Panel A. Willingness-to-pay default risk parameter $\gamma = 0.20$						
1: Before	1.020000	1.1165	0.00%	0.0203	0.0010	-13.1706	311.0446	-0.4803	—
2a: After (no SEAL)	—	—	—	—	—	—	—	—	—
2b: After, SEAL = 0.2	—	—	—	—	—	—	—	—	—
3: Max CE (no SEAL)	—	—	—	—	—	—	—	—	—
			Panel B. Willingness-to-pay default risk parameter $\gamma = 0.05$						
1: Before	1.020000	1.0307	17.55%	0.0337	0.0953	-1.4875	3.4568	-0.4828	—
2a: After (no SEAL)	0.987469	1.0539	1.84%	0.0505	0.0391	-5.0453	31.0385	-0.4556	294
2b: After, SEAL = 0.2	0.977643	1.0641	1.84%	0.0630	0.0430	-4.4276	26.0038	-0.4454	411
3: Max CE (no SEAL)	0.985308	1.0518	1.56%	0.0504	0.0361	-5.5332	37.1428	-0.4553	297

Note: The CE pickups for $\beta = 5$ are 399, 513, and 405, respectively.

Table 3:

Optimization outcomes for a single loan with an original LTV of 1.20. All other parameters are the same as in Table 2. "D" indicates that the loan is in default and no return is earned.

Case	LTV	Price	Pr(Def)	Mean	Stdev	Skew	Kurt	E[U(W)]	CE
Panel A. Willingness-to-pay default risk parameter $\gamma = 0.20$									
1: Before	1.200000	1.0548	37.39%	0.0565	0.1835	-0.4217	1.2705	-0.4957	—
2a: After (no SEAL)	1.137059	1.2081	0.91%	0.1653	0.0467	-6.1968	49.1991	-0.3709	1,560
2b: After, SEAL = 0.2	1.082747	1.1816	0.43%	0.1432	0.0360	-6.2857	59.5938	-0.3842	1,359
3: Max CE (no SEAL)	1.131110	1.2022	0.59%	0.1650	0.0383	-7.5355	73.5909	-0.3703	1,571
Note: The CE pickups for $\beta = 5$ are 1903, 1720, and 1930, respectively.									
Panel B. Willingness-to-pay default risk parameter $\gamma = 0.05$									
1: Before	1.200000	0.9000	100.00%	-D-	-D-	-D-	-D-	-0.5000	—
2a: After (no SEAL)	0.987469	1.0539	1.84%	0.1861	0.0391	-5.0453	31.0385	-0.3569	1,836
2b: After, SEAL = 0.2	0.977643	1.0641	1.84%	0.1986	0.0430	-4.4276	26.0038	-0.3498	1,956
3: Max CE (no SEAL)	0.985308	1.0518	1.56%	0.1860	0.0361	-5.5332	37.1428	-0.3568	1,839
Note: The CE pickups for $\beta = 5$ are 1813, 1929, and 1819, respectively.									

Table 4:

Probability of default and loan return distributions under falling or stagnating home values. Using the same loan parameters as in Table 3, we now compare pre- and post-restructuring loan return distributions over a one-year holding period under varying home price growth rates. For illustrative purposes, we consider the more conservative default recovery rate of $\phi = 0.9$. The certainty equivalents (CE) are computed relative to the base case loan before restructuring. For each case, the principal writedown is determined such that the certainty equivalent of the restructured investment relative to that of the original loan is maximized. "D" indicates that the loan is in default and no return is earned.

Case	LTV	Price	Pr(Def)	Mean	Stdev	Skew	Kurt	E[U(W)]	CE (bps)
Panel A. Home price growth rate $\mu = -0.04$									
When the willingness-to-pay default risk parameter is $\gamma = 0.20$:									
Before	1.200000	1.0548	89.55%	-0.1448	0.0951	2.8043	9.2264	-0.7017	—
After, no SEAL	1.052533	1.1500	0.56%	0.0890	0.0411	-6.1684	53.0809	-0.4244	2,859
After, SEAL=0.2	1.023471	1.1376	1.31%	0.0637	0.0543	-4.3976	27.0505	-0.4471	2,528
When the willingness-to-pay default risk parameter is $\gamma = 0.05$:									
Before	1.200000	0.9000	100.00%	-D-	-D-	-D-	-D-	-0.5000	—
After, no SEAL	0.924225	1.0093	2.20%	0.1134	0.0440	-4.0265	21.1978	-0.4057	1,101
After, SEAL=0.2	0.922378	1.0265	3.56%	0.1163	0.0546	-3.0786	13.1526	-0.4049	1,112
Panel B. Home price growth rate $\mu = 0$									
When the willingness-to-pay default risk parameter is $\gamma = 0.20$:									
Before	1.200000	1.0548	66.43%	-0.0631	0.1631	0.8179	1.7898	-0.6153	—
After, no SEAL	1.104286	1.1946	1.24%	0.1302	0.0541	-5.1935	35.1217	-0.3956	2,471
After, SEAL=0.2	1.060987	1.1692	1.24%	0.1071	0.0526	-4.7354	30.8962	-0.4120	2,220
When the willingness-to-pay default risk parameter is $\gamma = 0.05$:									
Before	1.200000	0.9000	100.00%	-D-	-D-	-D-	-D-	-0.5000	—
After, no SEAL	0.961942	1.0391	2.77%	0.1525	0.0472	-3.9314	19.5587	-0.3789	1,488
After, SEAL=0.2	0.952371	1.0494	2.77%	0.1593	0.0502	-3.5649	17.2173	-0.3747	1,552

Table 5:

Comparing return distributions over a one-year holding period on single-loan, two-loan, and 100-loan portfolios, where the initial $LTV = 1.02$, ability to pay is not an issue, and the willingness-to-pay default risk parameter is $\gamma = 0.05$. The input parameters are $H = 1$, $r_f = 0.02$, $T = 5$ years, recovery rate $\phi = 0.9$, growth rate $\mu = 0.04$, volatility $\sigma = 0.04$, default risk parameter $\gamma = 0.05$, and coupon rate $c = 0.04$ (the same configuration as used in Tables 2). We use $n = 100$ periods on the tree for loan pricing. Panel A presents the return distributions without loan restructuring. Panels B and C present the return distributions following a principle write-down such that the loan value is optimized. When restructuring entails shared appreciation, we set the strike $K = 1$. The certainty equivalents (CE) are computed relative to the base case single loan (before restructuring). In comparison to the single loan case where the barrier default probability is known in closed form, when there are many loans, the joint default distribution is simulated, and hence, the returns are not exactly at the mean of the single loans.

Corr(Z_i,Z_j)	N	Mean	Stdev	Skew	Kurt	E[U(W)]	CE(bps)
Panel A. Return distribution before restructuring: $LTV = 1.02$							
–	1	0.0337	0.0953	-1.4875	3.4568	-0.4828	—
0.7	2	0.0337	0.0660	-1.3372	4.0050	-0.4745	87
0.95	2	0.0336	0.0718	-1.4258	4.0812	-0.4760	71
0.7	100	0.0336	0.0588	-1.2553	3.9495	-0.4731	102
0.95	100	0.0339	0.0702	-1.4124	4.0736	-0.4753	79
Panel B. Return distribution after restructuring: $LTV = 0.978653$, no SEAL							
–	1	0.0464	0.0296	-6.3492	51.2446	-0.4581	265
0.7	2	0.0473	0.0239	-4.8672	34.7103	-0.4562	287
0.95	2	0.0479	0.0264	-5.3523	40.0717	-0.4564	285
0.7	100	0.0480	0.0199	-4.2894	29.3746	-0.4558	292
0.95	100	0.0478	0.0257	-5.2847	39.2778	-0.4563	286
Panel C. Return distribution after restructuring: $LTV = 0.977619$, SEAL = 0.2							
–	1	0.0630	0.0430	-4.4283	26.0203	-0.4454	411
0.7	2	0.0644	0.0350	-3.3485	17.9429	-0.4431	438
0.95	2	0.0644	0.0383	-3.6874	20.4037	-0.4434	434
0.7	100	0.0645	0.0300	-2.9327	15.3818	-0.4424	446
0.95	100	0.0645	0.0374	-3.6488	20.2732	-0.4432	437

Table 6:

Optimization outcomes for a single loan with no shared appreciation. The default recovery rate is $\phi = 0.9$ and ability-to-pay is now an issue, with mean income $\mu_i = 0.12$ and income uncertainty $\sigma_i = 0.06$, relative to a coupon rate of 0.04. The initial home value is normalized to $H = 1$, remaining loan maturity is $T = 5$ years, recovery rate $\phi = 0.9$, home price volatility is 4%, and the *original* coupon rate on the loan is 4%. We compare optimization outcomes using two different willingness-to-pay parameters: $\gamma = 0.20$ (Panel A), and $\gamma = 0.05$ (Panel B). We assume a riskless rate of $r_f = 0.02$, and we use $n = 100$ periods on the tree for loan pricing. Returns on the loan are computed assuming an investment horizon of $\tau = 1$ year and a home price growth rate of $\mu = 0.04$. We calculate the returns and expected utilities from holding this loan in each of the following cases: (1) before restructuring; (2) after writing down the debt to a level where the LTV is such that the loan value is optimized; (3) after writing down the coupon (but keeping the loan principal fixed) such that the annual payment is the same as in case 2; and (4) after writing down the *coupon* to a level such that the loan value is optimized. The overall probability of default as well as mean, standard deviation, skewness, and kurtosis of the return distribution with horizon τ are provided below. The certainty equivalents (CE) are computed relative to the base case loan before restructuring. "D" indicates that the loan is in default, and "—" indicates that coupon reduction does not lead to an improvement in the loan value, and hence no restructuring is undertaken.

Case	LTV	Cpn	Price	Pr(Def)	Mean	Stdev	Skew	Kurt	E[U(W)]	CE
Panel A. Willingness-to-pay default risk parameter $\gamma = 0.20$										
1: Before	1.200000	0.040000	1.0370	44.59%	0.0547	0.1690	-0.4117	1.2719	-0.4895	—
2: L writedown	1.137059	0.040000	1.1751	11.52%	0.1555	0.0448	-5.7668	44.2841	-0.3770	1,395
3: Cpn writedown	1.200000	0.037902	1.0334	44.09%	0.0509	0.1661	-0.4102	1.2718	-0.4917	−23
4: Max cpn writedown	1.200000	—	—	—%	—	—	—	—	—	—
Panel B1. Willingness-to-pay default risk parameter $\gamma = 0.05$										
1: Before	1.200000	0.040000	0.9000	100.00%	-D-	-D-	-D-	-D-	-0.5000	—
2: L writedown	0.996340	0.040000	1.0402	12.35%	0.1754	0.0493	-3.7163	17.0769	-0.3644	1,714
3: Cpn writedown	1.200000	0.033211	0.9000	100.00%	-D-	-D-	-D-	-D-	-0.5000	—
4: Max cpn writedown	1.200000	—	—	—%	—	—	—	—	—	—
Panel B2. Willingness-to-pay default risk parameter $\gamma = 0.05$										
1: Before	1.020000	0.040000	1.0185	25.25%	0.0340	0.0902	-1.4581	3.4040	-0.4807	—
2: L writedown	0.996340	0.040000	1.0402	12.35%	0.0517	0.0493	-3.7163	17.0769	-0.4560	268
3: Cpn writedown	1.020000	0.039072	1.0159	25.03%	0.0314	0.0889	-1.4564	3.4004	-0.4828	−22
4: Max cpn writedown	1.020000	—	—	—%	—	—	—	—	—	—

Table 7:

Optimization outcomes for a single loan with no shared appreciation. The default recovery rate is $\phi = 0.9$ and *willingness-to-pay* is no longer an issue (i.e., $\gamma = 100$) but ability-to-pay is, where we consider mean income and and income uncertainty ($\{\mu_i, \sigma_i\}$) of $\{0.02, 0.02\}$ (Panel A) and $\{0.04, 0.04\}$ (Panel B), relative to a coupon rate of 0.04. The initial home value is normalized to $H = 1$, remaining loan maturity is $T = 5$ years, recovery rate $\phi = 0.9$, home price volatility is 4%, and the *original* coupon rate on the loan is 4%. We assume a riskless rate of $r_f = 0.02$, and we use $n = 100$ periods on the tree for loan pricing. Returns on the loan are computed assuming an investment horizon of $\tau = 1$ year and a home price growth rate of $\mu = 0.04$. We calculate the returns and expected utilities from holding this loan in each of the following cases: (1) before restructuring; (2) after writing down the debt to a level where the LTV is such that the loan value is optimized; (3) after writing down the coupon (but keeping the loan principal fixed) such that the annual payment is the same as in case 2; and (4) after writing down the *coupon* to a level such that the loan value is optimized. The overall probability of default as well as mean, standard deviation, skewness, and kurtosis of the return distribution with horizon τ are provided below. The certainty equivalents (CE) are computed relative to the base case loan before restructuring. "—" indicates that reduction in principal does not lead to an improvement in the loan value, and hence no restructuring is undertaken.

Case	LTV	Cpn	Price	Pr(Def)	Mean	Stdev	Skew	Kurt	E[U(W)]	CE
Panel A. mean income $\mu_i = 0.02$, income uncertainty $\sigma_i = 0.02$										
1: Before	1.200000	0.040000	0.9335	91.92%	0.0371	0.0355	0.0133	3.0037	-0.4665	—
2: L writedown	1.114011	0.040000	0.9351	89.03%	0.0384	0.0343	0.0168	3.0061	-0.4653	13
3: Cpn writedown	1.200000	0.037134	0.9437	89.03%	0.0473	0.0340	0.0177	3.0067	-0.4573	100
4: Max cpn writedown	1.200000	0.005810	1.0626	25.74%	0.1539	0.0089	0.0925	7.1355	-0.3756	1,145
Panel B. mean income $\mu_i = 0.04$, income uncertainty $\sigma_i = 0.04$										
1: Before	1.200000	0.040000	1.0743	57.93%	0.0296	0.0196	0.0607	3.1653	-0.4722	—
2: L writedown	—	—	—	—%	—	—	—	—	—	—
3: Cpn writedown	—	—	—	—%	—	—	—	—	—	—
4: Max cpn writedown	1.200000	0.022601	1.0972	37.37%	0.0472	0.0124	0.0820	4.0649	-0.4562	174

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